Resolver

$$\boxed{\begin{array}{lll} u''_{xx} = u'_t & 0<x<\infty,\ t>0 & \\ u(0,t)=0 & t>0 & \\ u(x,0) = \begin{cases} 1 & 0<x<1 \\ 0 & x>1 \end{cases} & & = f(x) \end{array}}$$

T. seno: $\hat{U}_S(\omega,t) = \int_0^\infty u(x,t)\,\mathrm{sen}(\omega x)\,dx = -\frac{1}{2i}$ T. Fourier de extensión impar en x de $u(x,t)$

La ED:

$$-\omega^2 \hat{U}_S(\omega,t) = \hat{U}'_t(\omega,t)$$

$$\Rightarrow \hat{U}_S(\omega,t) = A(\omega).e^{-\omega^2 t}$$

en $t=0$: $\hat{U}_S(\omega,0) = A(\omega) = \int_0^\infty u(x,0)\,\mathrm{sen}(\omega x)\,dx = \hat{f}_S(\omega)$

$$\hat{U}_S(\omega,t) = \underbrace{\hat{f}_S(\omega)}_{-\frac{1}{2i}\text{ T. Fourier de extensión impar de } f} \cdot \underbrace{e^{-\omega^2 t}}_{\text{T. Fourier de } e^{-\frac{x^2}{4t}} \cdot \frac{1}{\sqrt{4t\pi}}}$$

$$-\frac{1}{2i}\hat{U}(\omega,t) = -\frac{1}{2i}\hat{f}(\omega).e^{-\omega^2 t} = -\frac{1}{2i}\hat{f}(\omega).\widehat{e^{-x^2/4t}\frac{1}{\sqrt{4t\pi}}}$$

$$\Rightarrow u(x,t) = \tilde{f}(x) * \frac{e^{-\frac{x^2}{4t}}}{2\sqrt{t\pi}} = \int_{-\infty}^{\infty} \tilde{f}(z)\frac{e^{-\frac{(x-z)^2}{4t}}}{2\sqrt{t\pi}}\,dz =$$

$\tilde{f}$: ext. impar de f

$$= \int_{-1}^{0} -\frac{e^{-\frac{(x-z)^2}{4t}}}{2\sqrt{t\pi}}\,dz + \int_0^1 \frac{e^{-\frac{(x-z)^2}{4t}}}{2\sqrt{t\pi}}\,dz =$$

$y = x-z$

$$u(x,t) = \int_x^{x+1} -\frac{e^{-\frac{y^2}{4t}}}{2\sqrt{t\pi}}\,dy + \int_{x-1}^{x} \frac{e^{-\frac{y^2}{4t}}}{2\sqrt{t\pi}}\,dy$$